Exercise 42

Find the derivative. Simplify where possible.

$$y = x\tanh^{-1}x + \ln\sqrt{1-x^2}$$

Solution

Take the derivative using the chain and product rules.

$$\begin{aligned}
y' &= \frac{d}{dx}\left(x\tanh^{-1}x + \ln\sqrt{1-x^2}\right) \\
&= \frac{d}{dx}\left(x\tanh^{-1}x\right) + \frac{d}{dx}\left(\ln\sqrt{1-x^2}\right) \\
&= \left[\frac{d}{dx}(x)\right]\tanh^{-1}x + x\left[\frac{d}{dx}(\tanh^{-1}x)\right] + \frac{1}{\sqrt{1-x^2}}\cdot\frac{d}{dx}\left(\sqrt{1-x^2}\right) \\
&= (1)\tanh^{-1}x + x\left(\frac{1}{1-x^2}\right) + \frac{1}{\sqrt{1-x^2}}\cdot\frac{1}{2}(1-x^2)^{-1/2}\cdot\frac{d}{dx}(1-x^2) \\
&= \tanh^{-1}x + \frac{x}{1-x^2} + \frac{1}{\sqrt{1-x^2}}\cdot\frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x) \\
&= \tanh^{-1}x + \frac{x}{1-x^2} + \frac{1}{\sqrt{1-x^2}}\cdot\frac{-x}{\sqrt{1-x^2}} \\
&= \tanh^{-1}x + \frac{x}{1-x^2} - \frac{x}{1-x^2} \\
&= \tanh^{-1}x
\end{aligned}$$